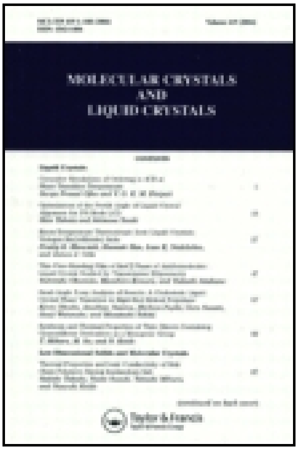

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Diffraction of Light on Liquid-Crystal Modulated Systems at Inclined Incidence

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Diffraction of Light on Liquid-Crystal Modulated Systems at Inclined Incidence†

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The subject of the present work is an experimental investigation of the diffraction for an inclined beam incidence to modulated LC structures forming various types of diffractional gratings. A number of new experimental data is obtained, analytical expressions are found for some of them. An effect was detected of asymmetrical extinction in odd orders of the diffraction at certain light beam incidence angles to the LC layer.

Liquid crystal (LC) modulated structures[1] are anisotropical diffractional gratings in which the phase modulation depth for the light wave is much larger than 1 rad, and the period amounts to dozens of wavelengths.[2] Such structures are of considerable interest in view of the development of new methods of information processing.[3] No theoretical investigation of the inverse diffraction problem was carried out until now for gratings of this type, and the direct problem was solved for the case where the phase shift modulation is close to 1 rad.[4–6] An interpretation of diffractograms obtained at the Williams domains for the normal light incidence to the grating was presented in Refs. 2 and 7. For the case of an inclined beam incidence to the LC grating the diffraction was studied only in the Raman–Nath approximation.[8]

The subject of the present work is an experimental investigation of the diffraction of an inclined beam incidence to modulated LC structures forming various types of diffractional gratings. A number of new experimental data is obtained, and analytical expressions are found for some of them. An

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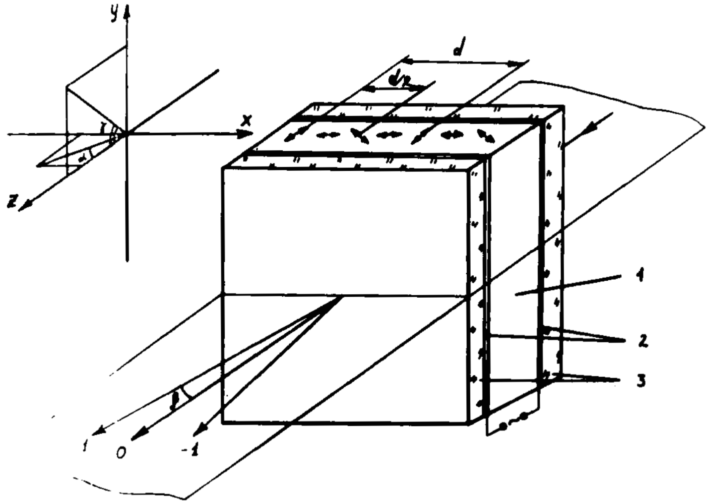

FIGURE 1 Sketch of the specimen and the coordinate system used; 1–the LC layer, 2–transparent conductive layers, 3–glass plates; β is the diffraction angle to the first order.

effect was detected of asymmetrical extinction in odd orders of the diffraction at certain light beam incidence angles to the LC layer.

The geometry of the experiment and the scheme of the diffractional grating are given in Figure 1. An inclined beam incidence to the grating is corresponding to non-zero angles α and γ.

The simplest way to observe the effect of subsequent extinction of the odd orders is to change α for $\gamma = 0$. It is established that the extinction of the n-th order (n is an odd number of any sign) is observed at an incidence angle α_n which equals one-half of the diffraction angle to this order, i.e. $\alpha_n = (1/2)\beta_n$. A number of diffractograms obtained for various incidence angles is presented in Figure 2. The incidence angles for the given diffractograms are chosen in such a way that the maxima from -7th to $+7$th order were extinguished (positions of these maxima are indicated by arrows).

The change in the diffractograms induced by variation of the angle γ is more complicated. At simultaneous rotation of the grating around the axes OX and OY, along with rotation of the diffraction surface which is not planar in this case, one also gets extinction of odd orders. However, in this case it is seen quite manifestly that the extinction is observed as a band, the position of which is determined by the magnitude of α and the slope to the plane XOZ depends on γ. For $\gamma \neq 0°$, $\alpha n = (1/2)\beta_n$ the band is parallel to the diffraction plane which is just the XOZ plane in this case. If the angles α and γ are of the same sign, the slope of the band is larger than 90°.

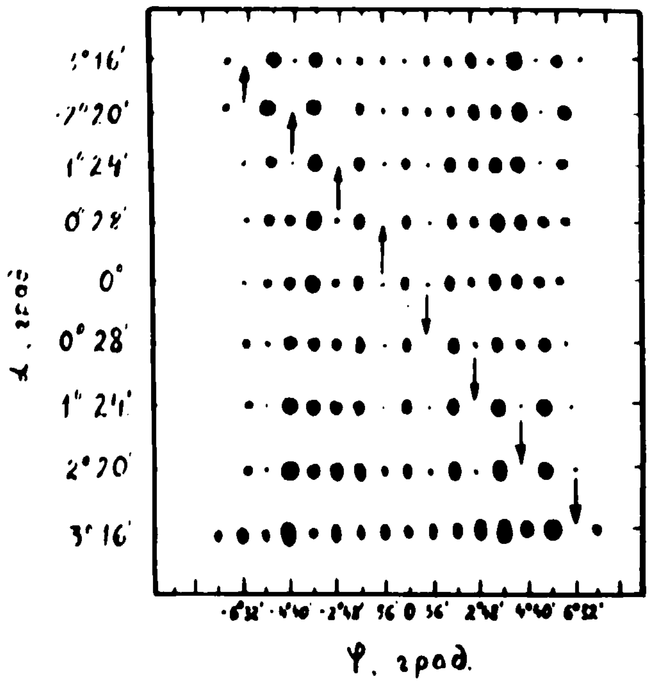

FIGURE 2 Diffractograms obtained for different values. Arrows indicate positions of extinguished diffraction orders.

If the signs are different, the slope is less than 90°. The band width also depends on γ, and falls with increasing γ. For $\gamma = 0°$ the band width is larger than the size of the diffraction spot, so it cannot be observed under these conditions.

Finally, we mention still another effect: for an inclined beam incidence ($\gamma = 0°$, $\alpha \neq 0°$) the diffractional picture appears for lower voltages at the specimen as compared with the normal incidence. Increasing the angle α we were able to reduce the threshold for appearance of the diffractional picture by 30%. Besides, if the diffractional picture was observed for some position of the specimen then an increase in α results in an increase in the number of diffractional maxima in the picture.

The effects mentioned above have been observed for all types of domains, except those where one has a twisting deformation (the domain sizes depend on the voltage).

The investigation of the α dependence of intensities of the diffractional maxima (for $\gamma = 0°$) has shown that near the angle of extinction of the n-th maximum its intensity is governed by the law

$$\mathcal{I}_n(\alpha) = \mathcal{I}_n(0)\frac{d^2}{n^2\lambda^2}\left(\alpha - \frac{1}{2}\frac{n\lambda}{d}\right) \qquad (1)$$

where $\mathcal{I}_n(0)$ is the intensity of the n-th maximum for the normal incidence, λ is the light wavelength, d is the LC-grating period.

For diffractional gratings prepared of isotropical materials, the asymmetry in the diffractional picture arising at an inclined beam incidence is due to space effects. These effects were considered exhaustively by S. M. Rytov[9] for the example of ultrasonic gratings. The reason for the asymmetry is the Bragg reflection, and the condition under which it is observable is

$$\lambda h/d^2 \simeq 1$$

where h is the grating width. For LC gratings, as a rule, d is about h by the order of magnitude and amount to 10^{-3} cm. Therefore, the above parameter is rarely higher than 0.1, so the asymmetry in the diffractional picture must not be manifest. Moreover, as it is seen in Figure 2, the extinction of the diffractional maxima takes place from the side opposite to that expected for the Bragg reflection. In other words, if the specimen is rotated by an angle $\alpha > 0$, a corresponding positive odd order is extinguished. Another feature where the case in view is different from the Bragg case is that the even orders are not extinguished for any magnitudes of the incidence angle.

One can explain both the effect of the extinction of the diffractional maxima and the decrease of the threshold for appearance of the diffraction, taking into account the fact that the LC gratings are optically uniaxial media, where the periodical variation of the optical properties is induced by periodical modulation of the direction of the optical axis. One may use either the Helfreich model, or the Penz model, describing the Williams domains near the threshold where they do arise.[10] Analyzing these models, we have observed that the structures described by them have the sliding reflection plane with the translation of $d/2$, coinciding with the plane X′OY′ (Figure 3). The effect of such a plane shows up in that the structure projected to this plane has a half period along the sliding component, which coincides with the OX′ axis (a–a′) in this case. On the other hand, since the grading is thick, i.e. its thickness is much larger than the light wavelength, this plane manifests itself only in the case where the beam trajectories in the LC layer have the symmetry of the structure. As it is seen from Figure 3 this condition holds for incidence angles which are equal to the Bragg angle for the corresponding diffractional maximum. Actually, if a part of the optical way AO is equivalent to O′B′, and OB is equivalent to A′O′ (we mean that the phase shifts for the respective segments are equal as there is no absorption) then the phase shift between the beams separated

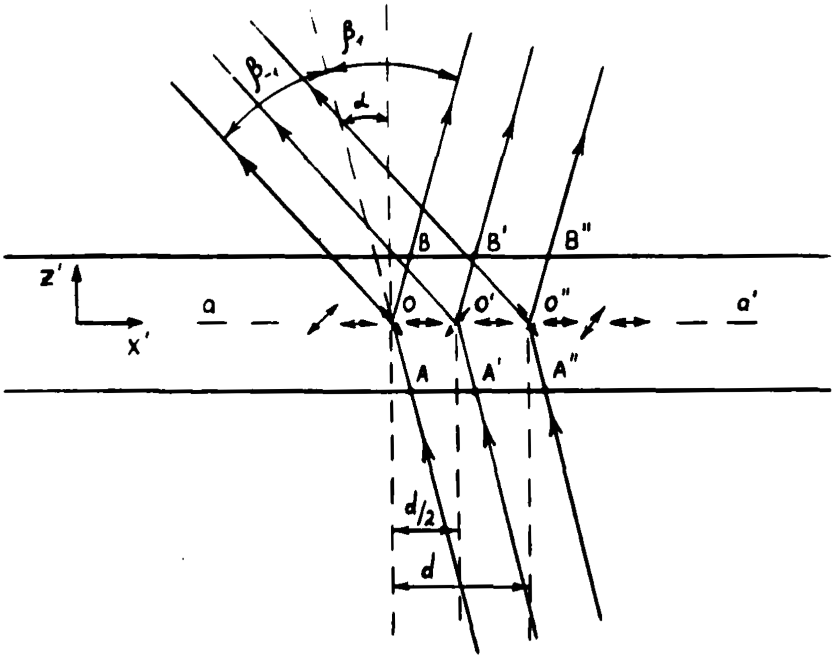

FIGURE 3 Cross section of the specimen by the XZ plane. The X'Y'Z' coordinate system is referred rigidly to the specimen.

by a distance of $d/2$, coming to a corresponding odd parameter, is ρ times an odd number (Figure 3). Consequently, if the beam incidence angle is one half of the diffraction angle to the n-th odd order, the corresponding maximum must extinguish. Meanwhile, even orders must not disappear, as for them the phase shift between the beams in view is ρ times an even number. It is this result that has been observed experimentally.

The experimentally observed dependence of the change in the intensity of the n-th odd maximum near the extinction angle can be explained based upon the interference law. In fact, the superposition of two coherent monochromatical waves of identical amplitudes, shifted by a phase of φ, leads to the following φ-dependence of the resulting wave intensity,

$$\mathcal{I}(\varphi) = 2A^2(1 + \cos\varphi) \tag{2}$$

Near the extinction angle, i.e. for $\alpha = (1/2)\beta_n + \Delta\alpha(\Delta\alpha \ll \beta_n)$ Eq. 2 acquires the form

$$\mathcal{I}(\varphi) = 4A^2\{1 + \cos[(2n + 1)\pi \pm \Delta\varphi]\} \simeq \frac{1}{2}A^2(\Delta\varphi)^2 \tag{3}$$

For sufficiently small values of α, corresponding to diffraction from the LC gradings ($\alpha < 20°$) one has

$$(\Delta\varphi)^2 = \left(\frac{2\pi}{\lambda} d\,\Delta\alpha\right)^2 \sim \left(\alpha - \frac{1}{2}\beta_n\right)^2$$

Thus the intensity of the diffractional maximum in a vicinity of its extinction angle has a quadratic dependence, the fact observed experimentally.

To explain the decrease in the diffraction threshold with rising angle α, it is more appropriate to use the one-dimensional Helfreich model, in which the refractive index is independent of the coordinate along OZ'. A deviation of the optical axis from the initial direction is given by the sine in this model,

$$\mathbf{n}(\chi') = \theta_0 \sin q\chi'$$

Here $\mathbf{n}$ is the optical axis (director), $q = \pi/d$ is the grading wave vector, θ_0 is the amplitude of deviation of the optical axis from the initial direction. For small θ_0 and normal beam incidence, the refractive index has also a sine dependence, but its period is one half of that in the above expression. This statement can be verified expanding the equation for the refractive indices of the uniaxial crystal in the series over powers of the slope near the direction of its semi-major axis, and retaining only the first non-vanishing term. The result is

$$n(\theta_0) = n_e\left[1 - \frac{1}{4}\theta_0^2 \frac{n_e^2 - n_0^2}{n_0^2}(1 - \cos 2q\chi')\right] \tag{4}$$

where n_e and n_0 are the semi-major and semi-minor axes of the ellipsoid, respectively. At the normal incidence near the diffraction threshold the diffractogram contains only the zero- and first-order maxima. If the specimen is rotated by an angle α (for $\gamma = 0$) a term linear in θ_0 appears in the expansion, and with the accuracy up to the second-order term it is given by

$$n(\alpha, \theta_0) = \frac{n_0 n_e}{\sqrt{n_e^2 - (n_e^2 - n_0^2)\cos^2\alpha}}\left\{1 - \frac{n_e^2 - n_0^2}{2[n_e^2 - (n_e^2 - n_0^2)\cos^2\alpha]}\right.$$
$$\cdot x\left[\theta_0 \sin qx' \sin 2\alpha + \frac{1}{y}\theta_0^2(1 - \cos 2qx')\right.$$
$$\left.\left.\cdot\left(\cos 2\alpha - \frac{3}{4}\frac{(n_e^2 - n_0^2)\sin^2 2\alpha}{[n_e^2 - (n_e^2 - n_0^2)\cos^2\alpha]}\right)\right]\right\} \tag{5}$$

In other words, for an inclined incidence the period for the refractive index conicides with the structure period. Therefore, a maximum corresponding to the period of orientation of the optical axis must be also observed in the diffractogram at the inclined incidence. Only zero- and first-order maxima are present indeed in the diffractogram at the diffraction threshold. It is remarkable that for the normal incidence the thresholds for appearance of the domain structure and diffraction coincide, while in the case of the inclined incidence no such correlation was found because it was too difficult to focus microscope to the inclined specimen. It was noticed, meanwhile, that the threshold for formation of the domains is also decreasing for larger α. The lowering of the voltage under which the appearance of the diffraction is detected, as well as the formation of the domains, taking place when α is increasing, is due to an increase in the "acting" modulation amplitude of the direction of optical axis. This is seen when one compares (4) and (5). Besides, an increase in the distance, covered by the light wave in the LC layer, which is an effect of the inclination, does also produce an increase in variation of the phase shift.

The described increase in the modulation of the phase shift with rise of the beam slope, when the voltage at the specimen is fixed, provides also with an explanation of the fact that the number of the diffraction maxima in the diffractogram is increasing with α.

The effect of extinction of odd orders has been observed also for voltages above the threshold when the grading structure was different from a sinusoid.[2] This is evidence that the domain structure contains the sliding reflection plane, though becomes nonlinear. This result is in agreement with conclusions based on calculation of the domain profile using diffractograms obtained from measurements at the normal incidence and observations of the LC flows in the domains.[11]

In conclusion, we present formulas exploited to calculate the domain size. For $\gamma = 0°$ and $\alpha \neq 0°$, the domain size is given by

$$2d \sin \frac{1}{2}\beta_n \cos\left(\alpha \pm \frac{1}{2}\beta_n\right) = n\lambda, \tag{6}$$

where the sign at $(1/2)\beta_n$ is determined by the sign of the beam slope. Another remarkable feature of the diffractograms is that for small slopes of the specimen the behaviors of the diffraction spectra are different in positive and negative orders. In this case the maxima of $+n$-th orders show a monotonous increase in their angular position, while there is a decrease in the $-n$-th orders. The angles β_{-n} acquire a minimal value for $\alpha = \arcsin(n\lambda/2d)$. For further increase in α a monotonous increase in β_{-n} takes place. Such a behavior of the diffractograms is in agreement with (6).

Now we turn back to the specimen inclinations in $\gamma(\alpha = 0°)$. As it was already mentioned, in this case the diffractogram is no longer planar

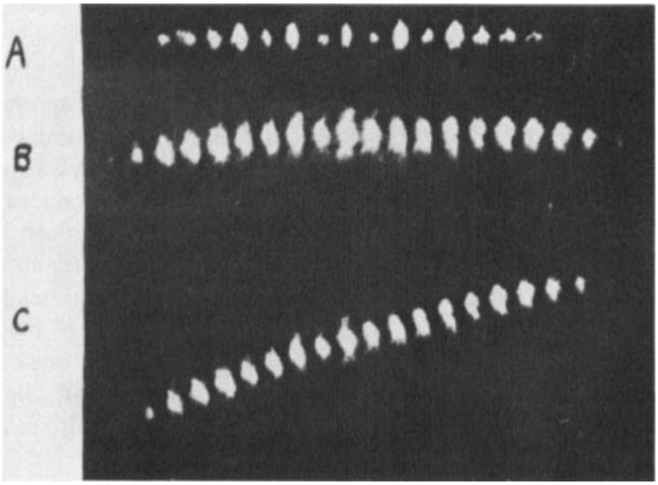

FIGURE 4 Diffractograms obtained for different values of α and γ. A–$\alpha = \gamma = 0°$; B–$\alpha = 0°$, $\gamma = 20°$; C–$\alpha = 30°$, $\gamma = 20°$.

(Figure 4B). The sign of the diffractogram curvature is opposite to the sign of γ. To explain the curvature of the diffraction surface one should have in mind that the incident beam has a finite width in the OY direction. A scheme illustrating the origin of the curvature of surface of the diffraction spectrum is given in Figure 5. There the diffractional grating L is perpendicular to the picture plane. The interference in view is between two beams lying in the ZOY plane and having a relative shift by a distance 1 along the OY axis. Directions of the beams undergoing diffraction to the $\pm n$-th orders are also shown. In this case an additional path-length difference $\Delta = AB - B'D$, that is zero for the normal incidence, appears for the beams indicated. It is seen from the picture that

$$\Delta = l tg\gamma - l\frac{\cos\gamma_n}{\cos\gamma},$$

where

$$\cos\gamma_n = \frac{B'D}{B'B} = \frac{B'E}{B'B}\cos\beta_n = \cos\gamma\cos\beta_n.$$

Since the diffractive maxima appear when the phase shift vanishes, to compensate the phase difference the beams must deviate from the diffraction plane P at an angle θ_n, given by the condition

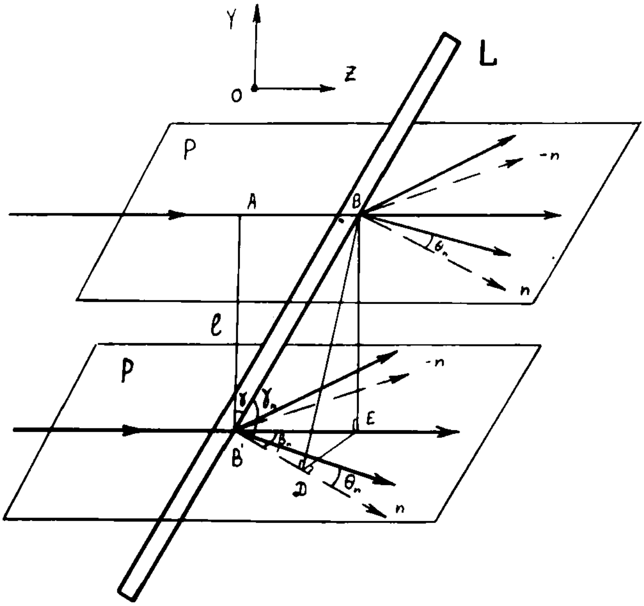

FIGURE 5 Schematic representation of the diffraction in the case where $\alpha = 0$ and $\gamma \neq 0$. The diffractional grating plane L is perpendicular to the picture plane.

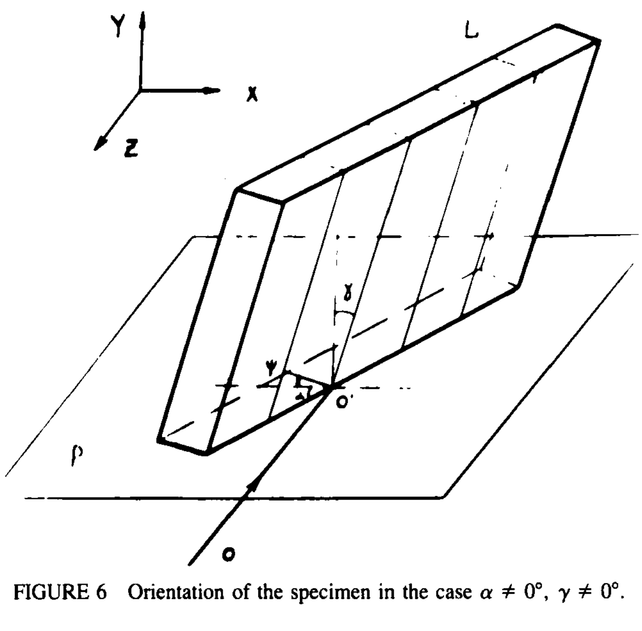

FIGURE 6 Orientation of the specimen in the case $\alpha \neq 0°$, $\gamma \neq 0°$.

$$l tg\gamma - \frac{l}{\cos\gamma}\cos(\gamma_n - \theta_n) = 0.$$

In both the cases described (either $\gamma = 0°$, or $\alpha = 0°$) the diffraction picture is not rotated around the direction of the incident beam (the OZ axis). In the general case of an inclined incidence such a rotation takes place. This phenomenon is due to the fact that in the case in view the grating period is inclined to the diffraction plane, corresponding to the normal incidence, by an angle $\psi = \gamma \sin\alpha$ (see Figure 6). Respectively, the diffraction plane is also rotated around the direction of the incident beam (OO′ in Figure 6) by the angle ψ (Figure 4C). In this situation all the properties, specific for the diffractogram in particular cases of the inclined incidence, are observed also in the general case.

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